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THE SET OF PERIODS FOR THE MORSE–SMALE DIFFEOMORPHISMS ON $\mathbb{T}^2$

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Abstract. In this paper, by using the Lefschetz zeta function, we characterize the set of periods of the Morse–Smale diffeomorphisms defined on the two–dimensional torus for every homotopy class. Our characterization distinguish between the class of orientation–preserving and orientation–reversing Morse–Smale diffeomorphisms. Moreover, we also characterize the minimal set of periods of the Morse–Smale diffeomorphisms.

Keywords. Morse–Smale diffeomorphism, Lefschetz number, zeta function, set of periods

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1 Introduction and statement of the main results

In dynamical systems and, particularly in the study of the iteration of self–maps defined on a given compact manifold (i.e. in discrete dynamical systems), the periodic behavior plays an important role. These last thirty years there was a growing number of results showing that certain simple hypotheses force qualitative and quantitative properties (like the set of periods) of a system. Perhaps the best known result in this direction is the paper entitle "Period three implies chaos" for the interval continuous self–maps which presents a weak version of the Sharkovskii's theorem, see [13].

One of the most useful results for proving the existence of fixed points, or more generally of periodic points for a continuous self–map f of a compact manifold, is the Lefschetz Fixed Point Theorem and its improvements, see for instance [2, 6, 8, 14, 15, 16]. For studying the periodic points of f it is convenient to use the Lefschetz zeta function $\mathcal{Z}_f(t)$ of f, which is a generating function of the Lefschetz numbers of all iterates of f. In Section 2 we provide a precise definition of all these notions.