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DETECTABILITY OF LINEAR IMPULSIVE SYSTEMS

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Abstract. This paper establishes the equivalence of three detectability-related properties for a class of linear impulsive systems. The first involves a gramian-based condition inspired by results for time-varying, discrete-time linear systems introduced decades ago. The second is the existence of an impulsive observer that yields exponentially stable impulsive error dynamics. Finally, the third property is exponential stability of an 'unobservable' subsystem identified from a decomposition of the original system derived from an invariant subspace that characterizes the set of unobservable states. A consequence of this analysis is that full state observability of a linear impulsive system is not necessary for asymptotic state estimation, a well-known fact for linear time-invariant systems. The main ideas of the paper are applied to the problem of synchronizing two Lorenz oscillators in which the driven system is realized as an impulsive observer for the drive system.

Keywords. linear impulsive systems; detectability; stability; state estimation; geometric methods.

AMS (MOS) subject classification: 93B07, 93B27, 93B50, 93C05, 93C30, 93D15

1 Introduction

Stabilizability and detectability are linear system properties of longstanding interest in connection with various feedback stabilization and estimation problems in control. In the linear time-invariant (LTI) case, stabilizability and detectability are often defined as the ability to achieve exponential stability via state feedback and output injection, respectively. Well-known Kalman-type decompositions for LTI systems lead to characterizations of stabilizability and detectability involving exponential stability of 'uncontrollable' and 'unobservable' subsystems, respectively. A generalization of these results to time-varying linear systems, for which the aforementioned decompositions are usually unavailable, is presented in [1] for the discrete-time case. In this work, *uniform stabilizability* (resp. *uniform detectability*), loosely defined as uniform positive definiteness of the reachability gramian (resp. observability gramian) along unstable trajectories, is shown to be necessary and sufficient for exponential stabilization via state feedback (resp. output injection). The recent analysis in [5] sharpens the underlying gain constructions