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GENERALISED DIFFUSIVE DELAY LOGISTIC EQUATIONS: SEMI-ANALYTICAL SOLUTIONS

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Abstract. This paper considers semi-analytical solutions for a class of generalised logistic partial differential equations with both point and distributed delays. Both one and two-dimensional geometries are considered. The Galerkin method is used to approximate the governing equations by a system of ordinary differential delay equations. This method involves assuming a spatial structure for the solution and averaging to obtain the ordinary differential delay equation models. Semi-analytical results for the stability of the system are derived with the critical parameter value, at which a Hopf bifurcation occurs, found. The results show that diffusion acts to stabilise the system, compared to equivalent non-diffusive systems and that large delays, which represent feedback from the distant past, act to destabilize the system. Comparisons between semi-analytical and numerical solutions show excellent agreement for steady state and transient solutions, and for the parameter values at which the Hopf bifurcations occur.

Keywords. semi-analytical solutions; reaction-diffusion-delay equations; logistic equation; Hopf bifurcations; distributed delay.

AMS (MOS) subject classification: 35,37,41.

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