

## SOLITARY WAVES AND THEIR STABILITY IN COLLOIDAL MEDIA: SEMI-ANALYTICAL SOLUTIONS

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**Abstract.** Spatial solitary waves in colloidal suspensions of spherical dielectric nanoparticles are considered. The interaction of the nanoparticles is modelled as a hard-sphere gas, with the Carnahan-Starling formula used for the gas compressibility. Semi-analytical solutions, for both one and two spatial dimensions, are derived using an averaged Lagrangian and suitable trial functions for the solitary waves. Power versus propagation constant curves and neutral stability curves are obtained for both cases, which illustrate that multiple solution branches occur for both the one and two dimensional geometries. For the one-dimensional case it is found that three solution branches (with a bistable regime) occur, while for the two-dimensional case two solution branches (with a single stable branch) occur in the limit of low background packing fractions. For high background packing fractions the power versus propagation constant curves are monotonic and the solitary waves stable for all parameter values. Comparisons are made between the semi-analytical and numerical solutions, with excellent comparison obtained.

**Keywords.** colloid, solitary wave, modulation theory, stability, bifurcation.

**AMS (MOS) subject classification:** 35, 78.

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