

EXACT SOLUTIONS FOR A CLASS OF RATHOLES IN HIGHLY FRICTIONAL GRANULAR SOLIDS

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Abstract. The governing equations for the stress field for Coulomb-Mohr granular solids are highly nonlinear and hence only a few analytical solutions are known. In the special case of highly frictional materials (where the internal angle of friction is ninety degrees), exact parametric solutions exist for plane and axially symmetric ratholes in the form of wedge, quadratic and cubic profiles. In this paper, we will show that in the highly frictional case, these governing equations are in fact linearizable. In particular, we consider power law ratholes superimposed upon a wedge hopper base. Two examples are considered where stress fields are determined as the hopper wedge/cone angle is varied.

Keywords. Highly Frictional Granular Solids, Ratholes, Diffusion Equation.

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