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A COMPLEX NETWORK ANALYSIS OF GRANULAR FABRIC EVOLUTION IN THREE-DIMENSIONS

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Abstract. Recent studies employing graph theoretic techniques from Complex Networks revealed the co-evolution of emergent minimal contact cycles and load-bearing force chains as mesoscopic structures that form the basic building blocks of self-organization. This study demonstrates previously observed trends for two-dimensional assemblages of circular discs to equally apply when network analysis is applied to data from three-dimensional systems comprising non-spherical particles. As previously reported for two-dimensional systems, the 3-cycles minimal contact cycle basis is both prevalent and persistent, providing support to force chains. In a new finding, the majority of those 3-cycles are arranged so that they share a common contact with the force chain column, transmitting nearly uniform normal contact force magnitudes at the three contacts. Persistent 3-cycles in the sample are absent in the region of strain localization in which force chains buckle, a finding that suggests a possible new structural indicator of failure and associated boundaries of flow.

Keywords. Granular media, complex networks, particles, stability, discrete element method.

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