

MATHEMATICAL STUDY OF BLOOD FLOW IN THE REAL MODEL OF THE RIGHT CORONARY ARTERY - BYPASS GRAFT SYSTEM

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Abstract. In this paper, we study the blood flow in the real model of the right coronary artery -bypass graft system under the real pulsatile condition. The human blood is assumed as an incompressible Non-Newtonian fluid and its flow is modeled by the unsteady state Navier-Stokes equations and the continuity equation. The effect of the existence and intensity of a stenosis in the right coronary artery on the blood flow behaviour is investigated. Numerical simulations are also undertaken to analyse the influence of the bifurcation angle of the bypass graft on blood pressure, velocity distribution and wall shear rate.

Keywords. Mathematical Modeling, Blood flow, Right Coronary Artery, Stenosis.

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