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USING LAPLACE TRANSFORM TO PRICE AMERICAN PUTS

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Abstract. This paper presents an efficient numerical approach, based on the Laplace transform, for pricing American puts. After the appropriate expressions of the optimal exercise price as well as the option price are found in the Laplace space based on the pseudo-steady-state approximation (see [26]), numerical inversions are performed to restore their corresponding values in the original time space. Among many numerical inversion techniques, we have found that three are most suitable for the functions arising from option pricing problems. Then, out of these three methods, we have also found that, through numerical experiments, the Stehfest method is the best, in terms of both numerical accuracy and computation efficiency. A great advantage of this numerical approach is its robustness of calculating the Greeks of an option.

Keywords. Laplace transform, moving boundary value problems, numerical Laplace inversion.

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