

NECESSARY CONDITIONS FOR MULTIOBJECTIVE OPTIMAL CONTROL PROBLEMS WITH STATE CONSTRAINTS

B. T. Kien¹, N.-C. Wong², and J.-C. Yao^{3*}

¹Department of Information and Technology,
National University of Civil Engineering, 55 Giai Phong, Hanoi, Vietnam;
email: kienbt@nuce.edu.vn

²Department of Applied Mathematics,
National Sun Yat-Sen University, Kaohsiung, Taiwan 804.
Email: wong@math.nsysu.edu.tw

³Center for General Education,
Kaohsiung Medical University, Kaohsiung, Taiwan 807.
Email: yaojc@kmu.edu.tw

*Corresponding author.

Abstract. Necessary conditions of optimality are derived for multiobjective optimal control problems with pathwise state constraints, in which the dynamics constrain is modeled as a differential inclusion. An example is also given to illustrate necessary optimality conditions. The result obtained extends the corresponding result in [27] and [31].

Keywords. Multiobject, optimal control, state constraint.

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