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DYNAMICS OF CONTINUOUS AND DISCRETE TIME SIV MODELS OF GONORRHEA TRANSMISSION

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Abstract. A continuous-time SIV (susceptible-infected-vaccinated) model of the transmission of Gonorrhea among homosexuals is analyzed. A basic reproduction number R_o is identified and it is shown that the disease-free equilibrium is globally asymptotically stable when $\mathbb{R}_o \leq 1$. It is also shown that this equilibrium is unstable when $\mathbb{R}_o > 1$ and there exists a globally asymptotically stable endemic equilibrium in this case. These results are obtained by using the theory of asymptotically autonomous dynamical systems to reduce progressively the dimension of the systems. A nonstandard discretization method is used to formulate a discrete time model and it is shown that this discrete-time model preserves some important dynamical characteristics of the continuous time model including the basic reproduction number. The results of the discrete-time model and the basic reproduction number do not depend on the discretization step size and are exactly the same as those of the continuous time model.

Keywords. Gonorrhea Transmission, Non-Standard Discretization Method, Asymptotically Autonomous, Basic Reproduction Number, Disease-Free Equilibrium, Endemic Equilibrium, Global Stability, Threshold Behavior.

AMS (MOS) subject classification: 92D30, 34D23.

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