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EXISTENCE AND MULTIPLICITY OF NONTRIVIAL NONNEGATIVE SOLUTIONS FOR A CLASS OF QUASILINEAR $P-$LAPLACIAN SYSTEMS

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Abstract. We consider the existence and multiplicity of nonnegative solutions for the following nonhomogeneous elliptic system

$$\begin{cases} -\Delta_p u + m_1(x)|u|^{p-2}u = \frac{1}{q} f_u(x,u,v) + g_u(x,u,v) & x \in \Omega, \\ -\Delta_p v + m_2(x)|v|^{p-2}v = \frac{1}{q} f_v(x,u,v) + g_v(x,u,v) & x \in \Omega, \end{cases}$$

with boundary conditions $|\nabla u|^{p-2}\frac{\partial u}{\partial n} = \frac{1}{r}\lambda h_u(x,u,v)$ and $|\nabla v|^{p-2}\frac{\partial v}{\partial n} = \frac{1}{r}\lambda h_v(x,u,v)$, where $\Omega \subset \mathbb{R}^N$ is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, Δ_p denotes the p-Laplacian operator defined by $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $1 \le q < p < r < \frac{pN}{N-p}$, $\lambda > 0$ and f, g and h are positively homogeneous C^1-functions of degrees q, 1 and r, respectively. By using the fibering maps and the Nehari manifold associated with the Euler functional for the problem, we prove that there exists λ^* such that for $\lambda \in (0, \lambda^*)$, the above problem has at least two nontrivial nonnegative solutions.

Keywords. critical points, nonlinear boundary value problems, quasilinear p- Laplacian system, fibering map, Nehari manifold.

AMS (MOS) subject classification: 35B38, 34B15, 35J92.

1 Introduction

In this paper we deal with the existence and multiplicity of nonnegative solutions for the following quasilinear elliptic system

$$\begin{cases} -\Delta_p u + m_1(x)|u|^{p-2}u = \frac{1}{q} f_u(x,u,v) + g_u(x,u,v) & x \in \Omega, \\ -\Delta_p v + m_2(x)|v|^{p-2}v = \frac{1}{q} f_v(x,u,v) + g_v(x,u,v) & x \in \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial n} = \frac{1}{r}\lambda h_u(x,u,v) & x \in \partial\Omega, \\ |\nabla v|^{p-2}\frac{\partial v}{\partial n} = \frac{1}{r}\lambda h_v(x,u,v) & x \in \partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0$, $1 \le q < p < r < p^*$ ($p^* = \frac{pN}{N-p}$ if $N > p$, $p^* = \infty$ if $N \le p$), $\Omega \subset \mathbb{R}^N$ is a bounded domain with the smooth boundary $\partial\Omega$, Δ_p denotes the p-Laplacian operator defined by $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $\frac{\partial}{\partial n}$ is the outer normal derivative and $m_1, m_2 \in C(\bar{\Omega}, \mathbb{R})$ are positive bounded functions. Also