Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **19 (2012)** 371-382
 http://www.watam.org

EXISTENCE AND UNIQUENESS OF MILD SOLUTION FOR ABSTRACT FRACTIONAL FUNCTIONAL DIFFERENTIAL EQUATIONS[3]

Xiao-Bao Shu[1], Yongzeng Lai[2] and Fei Xu[2]

[1]Department of Mathematics, Hunan University
Changsha 410082, P.R. China, sxb0221@163.com
[2]Department of Mathematics, Wilfrid Laurier University
Waterloo, Ontario, N2L 3C5, Canada
ylai@wlu.ca, fxu.feixu@gmail.com

Abstract. This paper deals with the existence and uniqueness of mild solutions of a class of fractional partial semilinear functional differential equations. A series of analytical results about the mild solutions are obtained by using fixed point method. Then we present an example to further illustrate the applications of the result.
Keyword Cauchy problem, Fractional abstract differential equation, Mild solutions.

1 Introduction

Recently fractional differential equations attracted the attention of many authors (see for instance [1 $\sim$ 20]). Many phenomena in engineering, physics, continuum mechanics, signal processing, electromagnetics, economics and science can be described efficiently by fractional order differential equations. In the same time, great progress in the theory of fractional differential equations has been achieved in recent years. In [14 $\sim$ 22] the authors proved the existence of mild solutions of abstract fractional partial differential equations by using semigroup theory, operator theory and fixed point theorem.

In [22], the authors investigated the existence of mild solutions of the following system

$$\begin{cases} D_t^\alpha u(t) = Au(t) + I^{1-\alpha} f(t, u_t), \ \ t \in I = [0, T], \\ u(t) = \varphi(t), \ \ t \in [-r, 0], \end{cases} \tag{1.1}$$

corrected the errors in paper [15] by Jaradat *et al*, and generalized some previous results.

The aim of this paper is to investigate the existence theorem for the solutions of fractional functional differential equations. We first define the mild solution of a fractional functional differential equation by using Laplace transform. By analyzing estimates of norms of operators $S_\alpha(t)$ and $T_\alpha(t)$,

[3]Research was partially supported by the Doctoral Fund of Ministry of Education of China (200805321017).