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A HYBRID METHOD FOR A FAMILY OF QUASI-NONEXPANSIVE AND LIPSCHITZ MULTI-VALUED MAPPINGS

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Abstract. Our purpose in this paper is first to study a mapping which is generated by a family of quasinonexpansive and Lipschitz multi-valued mappings. Further, using the shrinking projection method, we establish strong convergence theorems for solving fixed point problems of such mappings.

Keywords. Quasi-nonexpansive multi-valued mapping; Shrinking projection method; Common fixed point; Strong convergence.

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1 Introduction

Let D be a nonempty and convex subset of a Banach space E. The set D is called *proximinal* if for each $x \in E$, there exists an element $y \in D$ such that $\|x - y\| = d(x, D)$, where $d(x, D) = \inf\{\|x - z\| : z \in D\}$. Let $CB(D)$, $K(D)$ and $P(D)$ be the families of nonempty closed bounded subsets, nonempty compact subsets, and nonempty proximinal bounded subsets of D, respectively. The *Hausdorff metric* on $CB(D)$ is defined by

$$H(A, B) = \max\left\{\sup_{x\in A} d(x, B),\ \sup_{y\in B} d(y, A)\right\}$$

for $A, B \in CB(D)$.

A single-valued mapping $T : D \to D$ is called *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in D$. A multi-valued mapping $T : D \to CB(D)$ is called *nonexpansive* if $H(Tx, Ty) \leq \|x - y\|$ for all $x, y \in D$. An element $p \in D$ is called a *fixed point* of $T : D \to D$ (resp. $T : D \to CB(D)$) if $p = Tp$ (resp. $p \in Tp$). The fixed points set of T is denoted by $F(T)$.

The mapping $T : D \to CB(D)$ is called