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EXISTENCE OF HOMOCLINIC ORBITS FOR GENERAL PLANER DYNAMICAL SYSTEM OF LIÉNARD TYPE

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Abstract. In this work we study the existence of homoclinic orbits of the planer system of Liénard type

$$\dot{x} = K(y - F(x)), \quad \dot{y} = -g(x),$$

where K is strictly increasing and $K(\pm\infty) = \pm\infty$. We present sufficient and necessary conditions for this system to have a positive and a negative semiorbit which starts at a point on the curve $y = F(x)$ and approaches the origin without intersecting the x-axes. The conditions obtained are very sharp.

Keywords. Homoclinic orbit; Liénard system; planer system; differential equation; dynamical system

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1 Introduction

We consider the Liénard-type system

$$\dot{x} = K(y - F(x)), \quad \dot{y} = -g(x), \tag{1.1}$$

where $F(x)$, $g(x)$ and $K(u)$ satisfy the following assumption (see [10]):

(C$_1$) $F(x)$ and $g(x)$ are continuous on $\mathbb{R}$, with $F(0) = 0$ and $xg(x) > 0$ for $x \neq 0$, and $K(u)$ is continuously differentiable and strictly increasing with $K(0) = 0$ and $K(\pm\infty) = \pm\infty$,

which guarantees that the origin is the unique critical point of (1.1).

Stability of the zero solution, center problem, and oscillation of solutions of system (1.1) are studied in [10,18] under the following assumption: