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STOCHASTIC INFINITE DELAY LOTKA-VOLTERRA MODEL WITH MARKOVIAN SWITCHING

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Abstract. In this paper, we investigate a stochastic infinite delay Lotka–Volterra model with Markovian switching

$$dx(t) = \text{diag}\,(x_1(t), \ldots, x_n(t)) \left[\left(b(r(t)) + A(r(t))x(t) \right.\right.$$
$$\left.\left. + \; B(r(t)) \int_{-\infty}^{0} x(t+\theta)\, d\mu(\theta) \right) dt + \sigma(r(t))d\omega(t) \right],$$

where $\omega(t)$ is a standard Brownian motion and the initial data comes from an admissible space C_r. Taking into account both white and color environmental noises in the model, we discover that, under certain conditions, small white noises will suppress the population explosion in a finite time and guarantee the existence of a global positive solution. We further disscus the ultimate boundedness in mean of the solution. Both properties are natural requirements from the biological point of view.

Keywords. Brownian motion; Stochastic infinite delay equation; Generalized Itô formula; Markov chain; Ultimately bounded in mean.

1 Introduction

The Lotka–Volterra model plays an important role in modeling the population growth of certain species. The deterministic Lotka–Volterra model with infinite delay for n interacting species is generally described by the following integro-differential equations

$$\frac{dx(t)}{dt} = \text{diag}\,(x_1(t), \ldots, x_n(t)) \left[b + Ax(t) + B \int_{-\infty}^{0} x(t+\theta)\, d\mu(\theta) \right], \quad (1)$$

where

$$x = (x_1, \cdots, x_n)^T, \quad b = (b_1, \cdots, b_n)^T, \quad A = (a_{ij})_{n\times n}, \quad B = (b_{ij})_{n\times n}.$$