Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **19 (2012)** 199-208
 http://www.watam.org

GROWTH OF SOLUTIONS OF SOME NONLINEAR SECOND ORDER EQUATIONS

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Abstract. Recently, compared with complex differential equations, there has been an increasing renewed interest in complex difference equations and difference analogues of Nevanlinna theory. In this paper, we investigate a kind of nonlinear differential equations that involve difference, and obtain some estimations of the growth of solutions of these equations.

Keywords. Growth order, polynomial, entire functions, differential equations, diference.

AMS (MOS) subject classification: 34M10, 30D35.

1 Introduction and main results

In this paper a meromorphic function will mean meromorphic in the whole complex plane, and we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna Theory of meromorphic functions(e.g. see [10][11][12]). It is well known that the order $\sigma(f)$ and hyper-order $\sigma_2(f)$ of a meromorphic function f are defined by

$$\sigma(f) = \limsup_{r\to\infty} \frac{log\, T(r,f)}{log\, r},$$

$$\sigma_2(f) = \limsup_{r\to\infty} \frac{log\, log\, T(r,f)}{log\, r}.$$

Let η be a fixed, non-zero complex number, $\Delta f(z) = f(z+\eta) - f(z)$, and $\Delta^n f(z) = \Delta(\Delta^{n-1} f(z))$. In the study of the solutions of complex differential equations and the solutions of complex difference equations, the growth of a solution is an important property. For linear differential equations of the form

$$f^{(n)} + a_{n-1}(z) f^{(n-1)} + \cdots + a_0(z) f = 0, \tag{1.1}$$

where $a(z)$, $a_0(z)$, $\cdots$, $a_{n-1}(z)$ are entire functions, it is well known that any solutions of the equation (1.1) must be entire functions of finite order if $a(z)$, $a_0(z)$, $\cdots$, $a_{n-1}(z)$ are polynomials; and if one among the coefficients of the equation (1.1) is

[0]This work was supported by the NSF of Shandong Province, P. R. China(No.ZR2010AM030) and the NNSF of China(No. 11171013 & No.11041005). Corresponding author: Lianzhong Yang.