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POSITIVE SOLUTION FOR A FRACTIONAL EIGENVALUE PROBLEM

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Abstract. In this paper we consider the boundary value problem

$$\begin{cases} D^{\alpha}u(t) + \lambda a(t)f(u) = 0 & 0 < t < 1 \\ u(0) = u(1) = 0 \end{cases}$$

where $1 < \alpha \leq 2$ is a real number and D^{α} is the standard Riemann-Lioville differentiation. We determine the values of λ, so called eigenvalues, for which the boundary value problem has at least one positive solution under specific conditions on f.
Keywords. Fractional differential equation; Boundary value problem; Eigenvalue;
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1 Introduction

Fractional differential equations have been studied by many authors because of its applications in solving ordinary differential equations and also many applications in physics, mechanics, chemistry, and engineering. Positive solutions for ordinary differential equations and difference equations also have been considered by many authors, e.g. [1,4,6]. The major tool in finding positive solutions for both fractional and ordinary differential equations have been fixed point theorems and Leray-Shauder theory. Henderson and Wang [4] studied positive solution of the following boundary value problem

$$\begin{cases} u'' + \lambda a(t)f(u) = 0 & 0 < t < 1 \\ u(0) = u(1) = 0 \end{cases}$$

In this paper we improve this result by considering a boundary value problem associated to a fractional differential equation of the following form

$$\begin{cases} D^{\alpha}u(t) + \lambda a(t)f(u) = 0 \qquad 0 < t < 1 & (1.1) \\ u(0) = u(1) = 0 & (1.2) \end{cases}$$

where $1 < \alpha \leq 2$ is a real number and D^{α} is the standard Riemann-Lioville differentiation, we determine an interval for λ for which the fractional differential equation (1.1) with boundary condition (1.2) has at least one positive solution under the following conditions on f: