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EULER SOLUTIONS FOR DELAY DIFFERENTIAL EQUATIONS

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Abstract. In the delay differential equation

$$x' = f(t, x_t)$$

the symbol x_t can be defined in two ways. For example, if $x \in C[[t_0 - \tau, T], \mathbb{R}^n]$ and $\tau > 0$, then for each $t \in [t_0, T]$ (i) x_t is the graph of x on $[t - \tau, t]$ shifted to the interval $[-\tau, 0]$; (ii) x_t is the graph of x on $[t_0 - \tau, t]$.
The IVP

$$x' = f(t, x_t), \quad x_{t_0} = \phi_0$$

where $\phi_0 \in C_0 = C[[-\tau, 0], \mathbb{R}^n],\ \tau > 0$ and $f \in C[[t_0, T] \times C_0, \mathbb{R}^n]]$ relates to the case (i) which has been extensively studied and there is voluminous literature in this area. But the second case has not been considered to a large extent. In the second case the functional is known as Volterra operator which is obtained by t and the values of x(s) on the entire interval for each t, $t_0 - \tau \le s \le t$. The IVP $x' = f(t, x_t)$ involving the Volterra operator has not been studied till recently and that too it has been considered in the study of differential equations involving causal operators with memory. In this paper we construct the Euler solution for the above delay differential equation where the functional is a Volterra operator and further give criteria when an Euler solution becomes a solution.

Keywords. Delay differential equation, Euler solution.

1 Introduction

The x_t can be defined in several ways. For example if x is a function defined on some interval $[t_0 - h, T], \quad T > t_0$. Then for

In many physical phenomena the past history plays an important role along with the present state and hence an appropriate model of the phenomena will be one that involves past history. This is given by functional differential equations or differential equations involving delay. A general system of this type is described by the equation,

$$x' = f(t, x_t)$$

where f is a suitable functional. each $t \in [t_0, T]$,