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GLOBAL ATTRACTIVITY IN A HIGHER ORDER NONLINEAR DIFFERENCE EQUATION

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Abstract. We establish a sufficient condition for the global attractivity of solutions of the following higher order difference equation

$$x_{n+1} - x_n = p_n(f(x_{n-k}) - g(x_{n+1})),\ n = 0, 1, \cdots,$$

where $k \in \{0, 1, \cdots\}$, $f, g \in C[[0,\infty), [0,\infty)]$ with g nondecreasing, and $\{p_n\}$ is a nonnegative sequence. Several examples are also given to illustrate the applications of our main results.

Keywords: Global attractivity, Equilibrium, Positive Solutions, L-Lipschitz, Higher order difference equation
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1 Introduction

Consider the higher order difference equation

$$x_{n+1} - x_n = p_n(f(x_{n-k}) - g(x_{n+1})),\ n = 0, 1, \cdots, \tag{1.1}$$

where $k \in \{0, 1, \cdots\}$, $f, g \in C[[0,\infty), [0,\infty)]$ with g nondecreasing, and $\{p_n\}$ is a nonnegative sequence. Our aim in this paper is to study the global attractivity of positive solutions of Eq.(1.1).

With Eq.(1.1), we associate an initial condition of the form

$$x_{-k}, x_{-k+1}, \cdots, x_0 \in (0, \infty). \tag{1.2}$$

Observe that Eq.(1.1) can be written as

$$x_{n+1} + p_n g(x_{n+1}) = x_n + p_n f(x_{n-k}). \tag{1.3}$$

By noting that g is nondecreasing, it is easy to see that for each pair of $x_n, x_{n-k} \in (0,\infty)$, there is a unique $x_{n+1} \in (0,\infty)$ satisfying (1.3). Hence, with any initial condition of the form (1.2), Eq.(1.1) has a unique positive solution.