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EXACT TRAVELING WAVE SOLUTIONS FOR TWO MODELS OF PHASE TRANSITIONS DRIVEN BY CONFIGURATIONAL FORCES

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Abstract. This paper employs the hyperbolic tangent function expansion method to study the exact traveling wave solutions for two models of phase transitions driven by configurational forces. The first model consists of the partial differential equations of linear elasticity coupled to a quasilinear nonuniformly parabolic equation of second order, which describes the diffusionless phase transitions of solid materials and differs the Allen-Cahn equation only by the gradient term, this model is not conserved in time. The authors obtain the exact hyperbolic tangent function solutions which are solitary wave solutions for the first model. For the second model, which holds conservation in time and describes the phase transitions due to interface motion by interface diffusion, some standing wave solutions that only include the space variable are obtained.

Keywords. Exact traveling wave solutions; Phase transition models; Linear elasticity; Hyperbolic tangent function expansion method; Standing waves

AMS (MOS) subject classification: 35K65, 35C07

1 Introduction

Two main types of phase transformations in solid materials can be distinguished, diffusion dominated and diffusionless transformations. In this paper we study two models of phase transitions driven by configurational forces. The first one has diffusive interfaces and consists of the partial differential equations of linear elasticity coupled to a quasilinear nonuniformly parabolic equation of second order. It is derived in [1, 2] from a sharp interface model for diffusionless phase transitions and can be considered to be a regularization of that model. However, the integral over the solution is not conserved in time in the first model, so we continue to study the second model that the conservation holds for the solution. We firstly formulate these initial-boundary value problems in the three-dimensional case and reduce them to the one-dimensional case, then we mainly study the exact traveling wave solutions for this two models in one-dimension.

Let $\Omega \subset \mathbb{R}^3$ be an open set. It represents the material points of a solid body. The different phases are characterized by the order parameter $S(x, t) \in$