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RANDOM STABILITY OF A FUNCTIONAL EQUATION ASSOCIATED WITH INNER PRODUCT: A FIXED POINT APPROACH

Choonkil Park[1], Jung Rye Lee[2] and Dong Yun Shin[3]

[1]Department of Mathematics, Research Institute for Natural Sciences, Hanyang University, Seoul 133-791, Korea

[2]Department of Mathematics, Daejin University, Kyeonggi 487-711, Korea

[3]Department of Mathematics, University of Seoul, Seoul 130-743, Korea

Abstract. Th.M. Rassias [Bull. Sci. Math. **108** (1984), 95–99] proved that the norm defined over a real vector space V is induced by an inner product if and only if for a fixed positive integer l

$$2l\left\|\frac{1}{2l}\sum_{i=1}^{2l}x_i\right\|^2+\sum_{i=1}^{2l}\left\|x_i-\frac{1}{2l}\sum_{j=1}^{2l}x_j\right\|^2=\sum_{i=1}^{2l}\|x_i\|^2$$

holds for all $x_1,\cdots,x_{2l}\in V$. For the above equality, we can define the following functional equation

$$2lf\left(\frac{1}{2l}\sum_{i=1}^{2l}x_i\right)+\sum_{i=1}^{2l}f\left(x_i-\frac{1}{2l}\sum_{j=1}^{2l}x_j\right)=\sum_{i=1}^{2l}f(x_i), \tag{1}$$

whose solution is realized as the sum of an additive mapping and a quadratic mapping.

Using fixed point method, we prove the Hyers-Ulam stability of the functional equation (1) in random Banach spaces.

Keywords. random Banach space, fixed point, functional equation related to inner product space, Hyers-Ulam stability, quadratic mapping, additive mapping.

AMS (MOS) subject classification: 46S50, 46C05, 39B52, 47H10.

1 Introduction

The stability problem of functional equations originated from a question of Ulam[35] concerning the stability of group homomorphisms. Hyers[15] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki[2] for additive mappings and by Th.M. Rassias[28] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Th.M. Rassias theorem was obtained by Găvruta[11] by replacing the unbounded Cauchy difference by a general control function in the spirit of the Th.M. Rassias' approach.

The functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y)$$