Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **19 (2012)** 187-206

SEMICONCAVITY OF THE MINIMUM TIME FUNCTION FOR DIFFERENTIAL INCLUSIONS

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Abstract. In this paper we consider the Minimum Time Problem with dynamics given by a differential inclusion. We prove that the minimum time function is semiconcave under suitable hypotheses on the multifunction F.

Keywords. Differential Inclusion, Semiconcavity, Optimality, Minimum Time Function.

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Received March 2011; revised May 2011; revised August 2011; revised September 2011.

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