

## OPTIMAL CONTROL OF THE SWEEPING PROCESS

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**Abstract.** We formulate and study an optimal control problem for the sweeping (Moreau) process, where control functions enter the moving sweeping set. To the best of our knowledge, this is the first study in the literature devoted to optimal control of the sweeping process. We first establish an existence theorem of optimal solutions and then derive necessary optimality conditions for this optimal control problem of a new type, where the dynamics is governed by discontinuous differential inclusions with variable right-hand sides. Our approach to necessary optimality conditions is based on the method of discrete approximations and advanced tools of variational analysis and generalized differentiation. The final results obtained are given in terms of the initial data of the controlled sweeping process and are illustrated by nontrivial examples.

**Keywords.** sweeping process, optimal control, dissipative differential inclusions, variational analysis, generalized differentiation.

**AMS (MOS) subject classification:** 49J52, 49J53, 49K24, 49M235, 90C30

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