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MAXIMUM PRINCIPLE FOR INFINITE-HORIZON OPTIMAL CONTROL PROBLEMS WITH DOMINATING DISCOUNT

Sergey M. Aseev[1] and Vladimir M. Veliov[2]

[1]International Institute for Applied Systems Analysis
Schlossplatz 1, A-2361 Laxenburg, Austria
Steklov Mathematical Institute, Gubkina 8, 119991 Moscow, Russia

[2] Institute of Mathematical Methods in Economics
Vienna University of Technology, Argentinierstr. 8/E105-4, A-1040 Vienna, Austria

Corresponding author email: veliov@tuwien.ac.at

Abstract. The paper revisits the issue of necessary optimality conditions for infinite-horizon optimal control problems. It is proved that the normal form maximum principle holds with an explicitly specified adjoint variable if an appropriate relation between the discount rate, the growth rate of the solution and the growth rate of the objective function is satisfied. The main novelty is that the result applies to general non-stationary systems and the optimal objective value needs not be finite (in which case the concept of overtaking optimality is employed). In two important particular cases it is shown that the current-value adjoint variable is characterized as the unique bounded solution of the adjoint equation. The results in this paper are applicable to several economic models for which the known optimality conditions fail.

Keywords: infinite horizon, Pontryagin maximum principle, transversality conditions.

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