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PERIODIC SOLUTIONS IN A DISCRETE MODEL WITH DELAY

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Abstract. An essentially nonlinear difference equation with delay serving as a mathematical model of several applied problems is considered. Sufficient conditions for the existence of periodic solutions are derived.

Keywords. Essentially nonlinear difference equations with delay; one-dimensional multivalued maps; $p-$invariance and attractivity; existence of periodic solutions; applications in physiology, economics, and medicine.

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1 Introduction

This paper deals with a class of scalar essentially nonlinear difference equations with delay of the form

$$\mu\Delta x_n = F(x_{n-K}) - G(x_{n+1}), \tag{1}$$

where $\Delta x_n := x_{n+1} - x_n$ is the standard difference, $\mu > 0$ is a real parameter, and the positive integer K is a delay. Functions F and G are real-valued and continuous.

Our motivation to study this equation is multifold. Firstly, equation (1) can be viewed as a discrete version of the continuous time differential delay equation

$$\varepsilon\dot{x}(t) = F(x(t-\tau)) - G(x(t)). \tag{2}$$

The latter has recently found several important applications in physiology, economics, and other areas (see e.g. [11, 12, 13, 14] and further references therein). It appears that equation (2) can possess quite complicated dynamics, including existence of periodic solutions and chaotic dynamics. Its complex dynamics are well documented for the case of linear $G(x) = -bx$, $b > 0$, see e.g. [1, 10, 15, 16, 19]. There are only a few studies on equation (2) in the