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ASYMPTOTIC STABILITY ANALYSIS OF A STOCHASTIC VOLTERRA INTEGRO-DIFFERENTIAL EQUATION WITH FADING MEMORY

L.E. Shaikhet[1] and J.A. Roberts[2]

[1]Department of Higher Mathematics
Donetsk State University of Management, 163a Chelyuskintsev Street, 83015 Donetsk, Ukraine

[2]Department of Mathematics
University of Chester, Parkgate Road, Chester, England CH1 4BJ
Corresponding author email:j.roberts@chester.ac.uk

Abstract. We investigate the long term behavior of solutions to a stochastic Volterra integro-differential equation with a fading memory; the fading memory is represented by using a decaying exponential convolution kernel. We give sufficient conditions for asymptotic mean square stability of the solution.

In a similar spirit, we investigate the long term behavior of solutions to discrete analogues of the above continuous problem; our discrete analogues are based on the Euler-Maruyama scheme for stochastic differential equations and θ-methods for approximating the integral term. We give necessary and sufficient conditions for asymptotic mean square stability of the trivial solution, obtaining our results by means of the general method of Lyapunov functionals construction. We focus also on the geometric interpretations of our findings, such as the sizes of stability regions. This enables us to make some conclusions with regards to choosing an appropriate θ-method for obtaining numerical approximations.

Keywords. Volterra, Lyapunov functionals, stochastic, integro-differential equation, difference equation, numerical analysis

1 Introduction

It is well known that Volterra functional equations are used to model systems with memory; such applications arise in fields as diverse as population dynamics, nuclear reactor dynamics, viscoelasticity and mathematical finance. See, for example, [4], [10], [12] and [15] for further discussions of applications. Randomness has a variety of roles to play in the modelling process; whether it be to simulate the random effects observed in the real-world system (such as the random occurrence of natural disasters having an effect on a population) or whether it be to simulate a parameter in the model which is only known to a particular degree of accuracy (see [19]).