

PERIODIC SOLUTIONS FOR A CLASS OF INFINITE LATTICES WITH SEMILINEAR RESTORING FORCE AND BOUNDED-COUPLING ¹

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Abstract. In this paper, by using the topological degree method and some limiting arguments, we study the existence of periodic solutions for a class of one-dimensional chain with semilinear restoring force and bounded-coupling. An approach via time-map is used to investigate the existence of periodic solutions for the equations. In our case, some unbounded conditions about the damping term are considered.

Keywords. infinite lattices, bounded-coupling, semi-linear, Periodic solutions, time-map, Brouwer degree.

AMS (MOS) subject classification: 34C15, 34C25.

1 Introduction

In this paper, we are concerned with the 2π -periodic boundary value problem for the infinite system of boundedly-coupled second-order differential equations

$$u_i'' + f_i(u_i)u_i' + g_i(u_i) = p_i(t, u, u'), \quad i \in \mathbf{Z}, \quad (1.1)$$

$$u(2\pi) - u(0) = u'(2\pi) - u'(0) = 0, \quad (1.2)$$

where $u \in \mathbf{R}^Z$, $u' \in \mathbf{R}^Z$, $f_i, g_i : \mathbf{R} \rightarrow \mathbf{R}$ are continuous, $p_i : [0, 2\pi] \times \mathbf{R}^Z \times \mathbf{R}^Z$ are continuous functions and 2π -periodic for the first variant and, for each $i \in Z$, p_i depends only on a finite number of components of u and u' and satisfies

$$(p_0) \quad |p_i(t, u, u')| \leq P_i \in R_+.$$

By a solution of (1.1)-(1.2) we mean a solution u of (1.1) such that $u_i(0) = u_i(2\pi)$, $u_i'(0) = u_i'(2\pi)$ for each $i \in \mathbf{Z}$.

The problems of the existence of the periodic solutions of infinite systems have been studied in many papers with different methods for its physical backgrounds. For example, the classical variational techniques is used in [1]- [4] to obtain the existence of the periodic solutions to the autonomous conservative one-dimensional lattice of particles and, meanwhile, in [9][10], a

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