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NONLINEAR SCHRÖDINGER EQUATIONS VIA FIXED POINT PRINCIPLES

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Abstract. This paper deals with weak solvability of the Cauchy-Dirichlet problem for the perturbed time-dependent Schrödinger equation. We use the operator approach based on fixed point theorems and properties of norm estimation and compactness of the solution operator associated to the nonhomogeneous linear Schrödinger equation. Also applied previously by the second author to nonlinear heat and wave equations, our operator method provides a unified way for treating different types of nonlinear boundary value problems.

Keywords. Nonlinear Schrödinger equation, weak solution, nonlinear operator, fixed point.

AMS (MOS) subject classification: 35Q55, 47J35.

1 Introduction

This paper deals with weak solvability of the Cauchy-Dirichlet problem for the perturbed Schrödinger equation:

$$\begin{cases} u_t - i\Delta u = \Phi(u) & \text{in } \ \Omega \times (0,T) \\ u(x,0) = g(x) & \text{in } \ \Omega \\ u = 0 & \text{on } \ \partial\Omega \times (0,T). \end{cases} \tag{1.1}$$

Here $\Omega \subset \mathbf{R}^n$ is a bounded domain and Φ is a general nonlinear operator which, in particular, can be a superposition operator, a delay operator, or an integral operator. Specific Schrödinger equations arise as models from several areas of physics. The problem is a classical one (see [2-6] and [11]) and our goal here is to make more precise the operator approach based on abstract results from nonlinear functional analysis. More exactly, we shall precise basic properties, such as norm estimation and compactness, for the (linear) solution operator associated to the nonhomogeneous linear Schrödinger equation and we shall use them in order to apply the Banach and Schauder theorems to the fixed point problem equivalent to problem (1.1). A similar programme has been applied to discuss nonlinear perturbations of the heat and wave equations in [8-10].