

## AN AUGMENTED LAGRANGIAN BASED SHOOTING METHOD FOR THE OPTIMAL TRAJECTORY GENERATION OF SWITCHING LAGRANGIAN SYSTEMS

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**Abstract.** A shooting based optimization method for the determination of the optimal trajectories of switching Lagrangian systems (SLS) is presented and applied to the model of a highly nonlinear and nonsmooth system with variable structure. This model of the differential-drive robot and the optimization algorithm are explained in detail, in order to exemplify the connections between the SLS and the hybrid system terminology. Several features of the augmented Lagrangian (AL) technique and the measure-differential inclusion (MDI) modeling of Lagrangian systems are favorably combined to obtain a globally-convergent and easy-to-implement shooting method. The method enables to convert a mixed integer programming type multi-point boundary value problem, which is NP-Hard, into a standard two-point boundary value problem for SLS. The augmented Lagrangian based direct shooting method obtains an at least locally-optimal trajectory over all hybrid executions of a SLS without requiring gradient information.

**Keywords.** nonsmooth analysis, friction, hybrid system, switching system, complementarity, trajectory optimization, optimal control.

**AMS (MOS) subject classification:** 37J55, 37J60, 49M30, 49J24, 49J52, 49J53, 70E18, 70E60.

## 1 Introduction

In this work the numerical open-loop optimal control problem of a class of hybrid systems which is called switching Lagrangian systems is studied. Roughly speaking, a SLS is a hybrid mechanical system with a set of discrete modes, and for each discrete mode there exist a domain together with a cost function defined on it. The continuous state, which consists of positions and velocities of the SLS, evolves within one of the domains, and upon hitting certain boundaries a trigger may change the discrete mode. The modes of SLS differ on the basis of their active constraints. The set of active mechanical constraints at any instant determines the mode in which the SLS evolves. Each constraint is associated with a constraint force. Thus a typical evolution of the system is partitioned into a number of curves in distinctive