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ON THE ENERGY FUNCTIONS OF NEURAL NETWORKS WITH DELAY

Wenli Zhu[1] Jin Hu[2] Pingzhi Li[1] and Cuijuan Zhu[1]

[1]Department of Economics Mathematics
South Western University of Finance and Economics, Chengdu, 610074, China

[2]School of Science
Chongqing Jiaotong University, Chongqing, 400074, China

Corresponding author email: windyvictor@gmail.com

Abstract. In this paper, by means of energy function approach and the method of coefficient variation as well as inequality analysis, it is proved that a neural network with time delay is stable if and only if all the solutions of the network are bounded. Relationship between equilibrium points of the network and local minimum points of the energy function is discussed. We also establish the sufficient and necessary conditions of the existence and uniqueness of the equilibrium point of delayed neural networks. As an application, a neural network model is provided to find out all the eigenvectors of a real symmetric matrix with respect to the largest eigenvalue.

Keywords. Delayed cellular neural network(DCNN), Energy function, Equilibrium point, Stability, Minimum point

1 Introduction

Since L. O. Chua [3] proposed the theory and application of cellular neural network (CNN), due to its great potential of applications, CNN has attracted great interest of more and more people [8]. CNN, consisting of many units with the name cell, is accomplished in static image processing [4], while to handle dynamic image processing, we need delayed CNN (DCNN) [12]. CNN can be described by ordinary differential equations and DCNN should be described by ordinary differential equations with time delay. Since the time delay of the network is one of the key factors causing the instability of the neural network, it is very important to study the dynamical behavior of the neural network with time delay. Some criteria of stability are given by the method of Lyapunov functional in many literatures [2,14,15], while few papers provide the stability results with the energy function approach.

The DCNN considered in this paper is given in the following differential equations with time delays:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j)) + S_i \ (i = 1, 2, \cdots, n; t \geq 0) \quad (1)$$

where n is the number of neurons, $x_i(t)$ denotes the state variable of the ith neuron at time t, $f_j(x_j(t))$ denotes the output of the jth neuron at time t, $c_i > 0$, a_{ij}, b_{ij}, S_i are constants, a_{ij} denotes the connection weight of the jth neuron on the ith neuron at time t, b_{ij} denotes the connection weight of the jth neuron on the ith neuron at time $t - \tau_j$, S_i denotes the external input, τ_j is nonnegative constant and denotes time delay. System (1) claims that the state of the neuron at any time is not only affected by the output of the neuron at that time, but also by that before that time. The relationship between