

## STABILITY CRITERIA FOR CERTAIN NEUTRAL HIGH EVEN ORDER DELAY DIFFERENTIAL EQUATIONS

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**Abstract.** In this paper we study the asymptotic stability of the zero solution of neutral even order linear delay differential equations of the form

$$y^{(2m)}(t) + \alpha y^{(2m)}(t - \tau) = \sum_{j=0}^{2m-1} a_j y^{(j)}(t) + \sum_{j=0}^{2m-1} b_j y^{(j)}(t - \tau)$$

where  $a_j, b_j, \alpha \in (-1, 0) \cup (0, 1)$  are certain constants and  $m \geq 1$ . Here  $\tau > 0$  is a constant delay. In this paper, we obtain a necessary condition and obtain robust method of determining whether the zero solution is asymptotically stable. In proving our results we make use of Pontryagin's theory for quasi-polynomials.

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## 1 Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of the neutral delay differential equation

$$y^{(2m)}(t) + \alpha y^{(2m)}(t - \tau) = \sum_{j=0}^{2m-1} a_j y^{(j)}(t) + \sum_{j=0}^{2m-1} b_j y^{(j)}(t - \tau) \quad (1.1)$$

where  $\tau > 0$ ,  $a_j, b_j$  ( $j = 0, 1, \dots, 2n - 1$ ) and  $\alpha$  are constants and  $m \geq 1$ . There are numerous applications of neutral differential equations in scientific models such as of masses attached to an elastic bar [1], population growth [2] and more. An interesting application of neutral equations appears in [3] and involves an interplay between physical observation and simulation (called "real-time dynamic substructuring") for seismic testing. There are many studies of neutral equations mainly dealing with oscillations or sufficient conditions of stability of the zero solution See [4-8]. In these studies necessary