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EXISTENCE RESULTS FOR A CLASS OF NONLINEAR EQUATIONS

Silvia - Otilia Corduneanu

Department of Mathematics
Gh. Asachi Technical University of Iaşi, Iaşi, Romania
scorduneanu@yahoo.com

Abstract. In this paper we present two existence results for the Cauchy problem $du = A(u)dt + dg,\ u(0) = \xi$. We first suppose that $g : [0, a] \to \mathbb{R}^n$ is a function of bounded variation, $A : \mathbb{R}^n \to \mathbb{R}^n$ is a dissipative continuous function with $A(0) = 0$, $\xi \in \mathbb{R}^n$ and we prove that the above Cauchy problem has a unique solution $u \in L^\infty([0, a]; \mathbb{R}^n)$. In the second case we consider a locally closed set $\Sigma \subset \mathbb{R}^n$, we choose in a suitable manner the functions A, g and we propose a tangency condition which is sufficient for the existence of a solution with values in Σ.

Keywords. Nonlinear equation, function of bounded variation, dissipative function.

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1 Introduction

In [5] it is presented a method closely related to Euler method of polygonal lines, to approximate the solution of the Cauchy problem

$$\begin{cases} u' = f(u) \\ \\ u(0) = \xi, \end{cases}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a dissipative continuous function with $f(0) = 0$ and $\xi \in \mathbb{R}^n$. We adapt some steps of the technique proposed in [5] and we discuss a similar method for the following Cauchy problem

$$\begin{cases} du = A(u)dt + dg \\ \\ u(0) = \xi. \end{cases} \tag{1.1}$$

In this context $A : \mathbb{R}^n \to \mathbb{R}^n$ is a dissipative continuous function such that $A(0) = 0$, $g : [0, a] \to \mathbb{R}^n$ is a function of bounded variation and $\xi \in \mathbb{R}^n$.

Definition 1.1 *We say that a function* $u \in L^\infty([0, a]; \mathbb{R}^n)$ *is a solution for the Cauchy problem* (1.1) *if*