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ON A KIRCHHOFF EQUATION WITH BALAKRISHNAN-TAYLOR DAMPING AND SOURCE TERM

Nasser-eddine Tatar[1] and Abderrahmane Zaraï[2]

[1]King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Dhahran 31261, Saudi Arabia
E-mail: tatarn@kfupm.edu.sa
[2]Cheikh El Arbi Tébessi University
12002 Tébessa, Algeria
E-mail: zaraiabdoo@yahoo.fr

Abstract. In this paper we establish sufficient conditions yielding exponential decay of solutions for a Kirchhoff problem. The equation is subject to a damping of Balakrishnan-Taylor type and a nonlinear forcing term. Moreover, we prove a blow up in finite time result.

1 Introduction

The objective of this work is to study the following initial-boundary value problem

$$\begin{cases} u_{tt} - \Big(\xi_0 + \xi_1 \left\|\nabla u\left(t\right)\right\|_2^2 + \sigma\left(\nabla u\left(t\right), \nabla u_t\left(t\right)\right)\Big)\Delta u \\ +\Delta^2 u + \nu\Delta^2 u_t = \left|u\right|^p u \ in\ \Omega\times[0,+\infty) \\ u\left(x,0\right) = u_0\left(x\right),\ u_t\left(x,0\right) = u_1\left(x\right)\ in\ \Omega \\ u\left(x,t\right) = \frac{\partial u}{\partial \eta}\left(x,t\right) = 0\ on\ \Gamma \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary Γ. Here $u(t,x)$ is the transverse deflection of the beam. All the parameters ξ_0, ξ_1, σ, ν and p are assumed to be positive constants. When $\xi_1 = \sigma = \nu = 0$, and without $\Delta^2 u$, the equation in (1) reduces to a nonlinear wave equation which has been extensively studied and several results concerning existence and nonexistence have been established [3, 5, 6, 8, 9, 11]. When ξ_0,$\xi_1 \neq 0$, $\sigma = \nu = 0$, and without $\Delta^2 u$, the equation in (1) reduces to the well-known Kirchhoff equation which has been introduced in [10] in order to describe the nonlinear vibrations of an elastic string. The equation introduced by Kirchhoff was

$$\rho h \frac{\partial^2 u}{\partial t^2} = \left\{p_0 + \frac{Eh}{2L}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right\}\frac{\partial^2 u}{\partial x^2} + f,$$