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A NOTE ON BIFURCATION STRUCTURE OF RADIALLY SYMMETRIC STATIONARY SOLUTIONS FOR A REACTION-DIFFUSION SYSTEM III

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Abstract. In this paper, we discuss the local bifurcation structure on the radially symmetric stationary solution of a certain reaction-diffusion system. To do this, we shall employ the comparison principle, and estimate the integral for the eigenfunction of the linearized operator around the constant solution.

Keywords. bifurcation structure, radially symmetric stationary solution, comparison principle.

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1 Introduction

To understand the mechanism of phenomena which appear in various fields, we often employ the reaction-diffusion system

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u} = \varepsilon\, D\, \Delta\mathbf{u} + \mathbf{f}(\mathbf{u}), \qquad x \in \Omega, \qquad t > 0, \\ \frac{\partial}{\partial \nu}\mathbf{u} = \mathbf{0}, \qquad x \in \partial\Omega, \qquad t > 0 \end{cases} \tag{1.1}$$

with suitable initial condition, and discuss the existence and stability of stationary solutions for the system, where $\mathbf{u} \in \mathbb{R}^N$, $\varepsilon > 0$, D is a diagonal matrix whose elements are positive, $\mathbf{f} : \mathbb{R}^N \to \mathbb{R}^N$ is a smooth function in $\mathbf{u}$, Ω is a bounded domain in $\mathbb{R}^\ell$ with smooth boundary $\partial\Omega$, and $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative on $\partial\Omega$.

When $N = 1$ holds, it is well-known that for suitable $\mathbf{f}(\mathbf{u})$, the global attractor $\mathcal{A}$ of (1.1) is represented as $\mathcal{A} = \bigcup_{\mathbf{e}\in E} W^u(\mathbf{e})$, where E is the set of stationary solutions of (1.1), and $W^u(\mathbf{e})$ is an unstable manifold of (1.1) at $\mathbf{u} = \mathbf{e}$ (for example, see Chapter 4 in Hale [2]). This fact suggests that to understand the precise long-time behavior of solutions for (1.1), one important problem is to seek out all stationary solutions of (1.1) and investigate their