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SECOND-ORDER NONLOCAL IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE AND OPTIMAL CONTROLS[1]

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Dedicated to Professor N.U. Ahmed on the occasion of his 75th birthday

Abstract. This paper is concerned with a class of second-order nonlocal nonlinear impulsive integro-differential equations of mixed type and corresponding optimal control problem. Introducing reasonable mild solution we prove the existence of mild solution. Then the existence of optimal controls for a Lagrange problem of systems governed by second-order nonlocal nonlinear impulsive integro-equations of mixed type is also presented. An example is given for demonstration.

Keywords. Second-order equation, nonlocal condition, impulse, integral operator of mixed type, mild solution, optimal control, Existence.

AMS (MOS) Subject Classification. 34G20, 34K30, 35B37, 93C25.

1 Introduction

In this paper we consider a class of second-order nonlocal nonlinear impulsive integro-differential equation of mixed type

$$\begin{cases} \ddot{x}(t) = A\dot{x}(t) + f(t, x(t), \dot{x}(t), (Gx)(t), (Hx)(t)), & t \in (0, T] \setminus \Theta, \\ x(0) = x_0, \qquad \Delta x(t_i) = J_i^0(x(t_i), \dot{x}(t_i)), & t_i \in \Theta, \\ \dot{x}(0) = x_1 + \varphi(x, \dot{x}), \qquad \Delta \dot{x}(t_i) = J_i^1(x(t_i), \dot{x}(t_i)), & t_i \in \Theta, \end{cases} \tag{1.1}$$

and corresponding optimal control problem. Here A is the infinitesimal generator of C_0-semigroup in a Banach space X, G, H are nonlinear integral operators given by

$$(Gx)(t) = \int_0^t k(t,\tau) g(\tau, x(\tau), \dot{x}(\tau)) d\tau, \ (Hx)(t) = \int_0^T m(t,\tau) h(\tau, x(\tau), \dot{x}(\tau)) d\tau.$$

J_i^0, $J_i^1 (i = 1, 2, \cdots, n)$ are nonlinear maps, and $\Delta x(t_i) = x(t_i + 0) - x(t_i)$, $\Delta \dot{x}(t_i) = \dot{x}(t_i + 0) - \dot{x}(t_i)$ which represent the jumps in the state x and