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SOLUTIONS FOR HEMIVARIATIONAL AND VARIATIONAL–HEMIVARIATIONAL INEQUALITIES

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Dedicated to Professor N.U. Ahmed on the occasion of his 75th birthday

Abstract. We consider nonlinear elliptic problems with unilateral constraints (hemivariational inequalities and variational–hemivariational inequalities). Using methods and techniques from nonsmooth critical point theory, we prove existence and multiplicity theorems when the linear part of the problem is indefinite. Our results extend in different ways earlier works in the literature.

Keywords. Nonsmooth critical point theory, unilateral constraints, C-condition, local linking, locally Lipschitz functional.

AMS (MOS) subject classification:35J50, 35J85.

1 Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a C^2-boundary $\partial\Omega$ and let $K \subseteq H_0^1(\Omega)$ be a closed convex cone (i.e., a closed convex set such that $\lambda K \subseteq K$ for all $\lambda \geq 0$). In this paper, we study the following two elliptic problems with a nonsmooth potential :

$$\left\{ \begin{array}{l} -\triangle u(z) + \alpha(z)u(z) \in \partial F\big(z, u(z)\big) \text{ in } \Omega, \\ x\big|_{\partial\Omega} = 0, \ \alpha \in L^{N/2}(\Omega), \ N \geq 3, \end{array} \right\} \text{ and } \tag{1}$$

$$\left\{ \begin{array}{l} \int_\Omega (Du, Dy - Du)_{\mathbb{R}^N} \mathrm{d}z + \int_\Omega \alpha u(y-u)\mathrm{d}z \geq \int_\Omega f(y-u)\mathrm{d}z \ \forall\, y \in K, \\ \text{with } u \in K, f(z) \in \partial F\big(z, u(z)\big) \text{ a.e. in } \Omega, \ \alpha \in L^{N/2}(\Omega), \ N \geq 3. \end{array} \right\} \tag{2}$$

In both problems the potential function $F(z,x)$ is only locally Lipschitz and in general nonsmooth in the x-variable. By $\partial F(z,x)$ we denote the generalized subdifferential in the sense of Clarke [3] of the function $x \longrightarrow F(z,x)$. Problem (1) is a "hemivariational inequality". Hemivariational inequalities were introduced to treat problems in mechanics and engineering where the relevant energy functionals are neither convex nor smooth (the so-called superpotentials). Many such applications can be found in the book of Naniewicz–Panagiotopoulos [13]. Another interesting feature of problem (1) is that the function $\alpha \in L^{N/2}(\Omega)$ may change sign. So, the