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EXISTENCE OF SOLUTIONS TO NONLINEAR SECOND ORDER EVOLUTION INCLUSIONS WITHOUT AND WITH IMPULSES

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Dedicated to Professor N.U. Ahmed on the occasion of his 75th birthday

Abstract. In this paper we investigate the existence of solutions to two classes of second order evolution inclusions with the Volterra integral operator. First we prove a result on unique solvability of the Cauchy problem without impulses by combining surjectivity theorem for multivalued pseudomonotone operators and the Banach Contraction Principle. Then, an existence result is provided for inclusions with impulses. These results are obtained in the framework of an infinite dimensional evolution triple of spaces.

Keywords. Evolution inclusion, pseudomonotone, multifunction, Volterra memory operator, impulsive, hemivariational inequality.

1 Introduction

An important number of problems arising in Mechanics, Physics and Engineering Science lead to mathematical models expressed in terms of nonlinear inclusions. For this reason the mathematical literature dedicated to this field is extensive and the progress made in the last decades is impressive. It concerns both results on the existence, uniqueness, regularity and behavior of solutions for various classes of nonlinear inclusions as well as results on numerical approach of the solution for the corresponding problems.

In this paper we study the problem of existence of solutions to the second order evolution inclusions with the Volterra memory operator. We formulate two Cauchy problems for evolution inclusions without and with impulses in the framework of evolution triple of spaces. The Cauchy problem for the evolution inclusion without impulses is of the form

$$\begin{cases} u''(t) + A(t, u'(t)) + B(t, u(t)) + \displaystyle\int_0^t C(t-s)u(s)\, ds + \\ \qquad\qquad + F(t, u(t), u'(t)) \ni f(t) \quad \text{a.e. } t \in (0, T), \\ u(0) = u_0, \quad u'(0) = v_0, \end{cases} \tag{$*$}$$