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STRONG REGULARITY OF TIME AND NORM OPTIMAL CONTROLS

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Dedicated to Professor N.U. Ahmed on the occasion of his 75th birthday

Abstract. Pontryagin's maximum principle in its infinite dimensional version provides (separate) necessary and sufficient conditions for both time and norm optimality for the system $y' = Ay + u$ (A the infinitesimal generator of a strongly continuous semigroup); in particular it provides a costate $z(t)$ for every time or norm optimal control $\bar{u}(t)$ hitting a target $\bar{y} \in D(A)$. This paper shows that for the right translation semigroup the same condition on $\bar{y}$ guarantees that $z(T) \in E^*$, which in turn implies continuity of optimal controls in the entire control interval $[0, T]$.

Keywords. Time optimality, norm optimality, Pontryagin's maximum principle, costate, smoothness of optimal controls.

AMS (MOS) subject classification: 93E20, 93E25.

1 Introduction

We study the control system

$$y'(t) = Ay(t) + u(t)\,, \quad y(0) = \zeta \tag{1.1}$$

with controls $u(\cdot) \in L^\infty(0, T; E)$, where A is the generator of a strongly continuous semigroup $S(t)$ in a Banach space E. We look at two optimal control problems for (1.1). One is the *norm optimal* problem, where we drive the initial point ζ to a point target,

$$y(T) = \bar{y} \tag{1.2}$$

in a fixed time interval $0 \le t \le T$ minimizing $\|u(\cdot)\|_{L^\infty(0,T;E)}$. The second is the *time optimal* problem, where we drive to the target with a bound on the norm of the control (say $\|u(\cdot)\|_{L^\infty(0,T;E)} \le 1$) in optimal time T. The *solution* or *trajectory* of (1.1) is given by the variation-of-constants formula

$$y(t) = y(t, \zeta, u) = S(t)\zeta + \int_0^t S(t - \sigma)u(\sigma)d\sigma \tag{1.3}$$