Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **18 (2011)** 513-523
 http://www.watam.org

THE HYBRID MAXIMAL MONOTONICITY WITH A NEW APPROACH TO GENERAL LINEAR CONVERGENCE ANALYSIS

Ram U. Verma

International Publications
3433 Mission Bay Blvd, Suite 331
Orlando, Florida 32817, USA

verma99@msn.com

Abstract. First a hybrid notion of *relative maximal monotonicity* is introduced and then it is applied to the approximation solvability of a general class of variational inclusion problems, while generalizing Rockafellar's theorem (1976) on linear convergence using the generalized proximal point algorithm in a real Hilbert space setting. The linear convergence analysis achieved in the case of Rockafellar based on the inexact proximal point algorithm is limited in the sense that it is achieved under the Lipschitz continuity at 0 of the inverse of the set-valued mapping involved, but later among several researchers, Xu (2002) modified the inexact proximal point algorithm still using the classical resolvent and established the general linear convergence, though the inexact proximal point algorithm does have similarities with the relaxed proximal point algorithm introduced and studied by Eckstein and Bertsekas (1992). In the present communication, a general convergence is achieved based on the new notion of the relative hybrid maximal monotonicity and corresponding inexact proximal point algorithm, but our construction breaks down for the maximal monotonicity and corresponding classical resolvent.

AMS (MOS) Mathematics Subject Classifications: 49J40, 47H10, 65B05
Key Words: Variational inclusion problems, Maximal monotone mapping, Relatively hybrid maximal monotone mapping, Generalized resolvent operator.

1. Introduction

Let X be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and with the norm $\| \cdot \|$ on X. We consider the inclusion problem: find a solution to

$$0 \in M(x), \tag{1}$$

where $M : X \to 2^X$ is a set-valued mapping on X.

Rockafellar [5, Theorem 2] discussed general convergence of the proximal point algorithm in the context of solving (1), by showing for M maximal monotone, that the sequence $\{x^k\}$ generated for an initial point x^0 by the proximal point algorithm

$$x^{k+1} \approx P_k(x^k)\,, k \geq 0, \tag{2}$$