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VARIATIONAL RELATION PROBLEMS IN LOCALLY CONVEX SPACES

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Abstract. Let A, B and Y be nonempty sets, $S_1 : A \rightrightarrows A$, $S_2 : A \rightrightarrows B$, $T : A \times B \rightrightarrows Y$ be set-valued mappings with nonempty values and $R(a, b, y)$ be a relation linking elements $a \in A, b \in B$ and $y \in Y$. In [1] Luc established existence theorems for solutions of the following problem: find $\bar{a} \in A$ such that $\bar{a}$ is a fixed point of S_1 and $R(\bar{a}, b, y)$ holds for all $b \in S_2(\bar{a})$ and $y \in T(\bar{a}, b)$. In this paper the same problem is investigated in locally convex Hausdorff topological vector spaces. Significant particular cases (quasivariational inclusion problems, quasivariational intersection problems, quasioptimization problems) will be also discussed.
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1 Introduction

Let A, B and Y be nonempty sets, $S_1 : A \rightrightarrows A$, $S_2 : A \rightrightarrows B$, $T : A \times B \rightrightarrows Y$ be set-valued mappings with nonempty values and $R(a, b, y)$ be a relation linking elements $a \in A, b \in B$ and $y \in Y$. In a general setting R is a subset of the product space $A \times B \times Y$. In practice, it is often given by a system of inequalities of real functions or a system of inclusions or intersections of set-valued mappings on $A \times B \times Y$. The following variational relation problem was recently introduced by Luc [1] (see also Khanh and Luc [2], Lin and Wang [3], Lin and Ansari [4], Luc et al. [5] for further studies) as a model for many problems in optimization, equilibrium theory, variational inclusions or variational inequalities:

(VR) Find $\bar{a} \in A$ such that

(i) $\bar{a} \in S_1(\bar{a})$;

(ii) $R(\bar{a}, b, y)$ holds for all $b \in S_2(\bar{a})$ and $y \in T(\bar{a}, b)$.

In [1] two existence results are established for solutions of the variational relation problem (VR) when $A = B$ is a nonempty compact convex set in a