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# ON A CLASS OF BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

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**Abstract.** In this paper we shall establish a new theorem on the existence and uniqueness of the adapted solution to a backward stochastic differential equation under some weaker conditions than the Lipschitz one. The extension is based on the Athanassov non-lipschitz condition for ordinary differential equations. Also, some stability properties of the solutions are given..

**Keywords.** backward stochastic differential equations, non-lipschitz conditions, adapted solutions, global solutions, stability properties.

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# 1 Introduction

A backward differential equation (see for example in [13]) which appears in the optimal stochastic control is the following:

$$x(t) + \int_t^1 f(s, x(s), y(s))ds + \int_t^1 g(s, x(s), y(s))dW(s) = X, \quad 0 \le t \le 1$$

where $\{W(t),\ 0 \le t \le 1\}$ is a Brownian motion defined on the probability space $(\Omega, \mathcal{F}, P)$ with the natural filtration $\{\mathcal{F}_t,\ 0 \le t \le 1\}$ and $X$ is a given $\mathcal{F}_1$-measurable random variable such that $E|X|^2 < \infty$. In the field of control, we usually regard $y(\cdot)$ as an adapted control and $x(\cdot)$ as the state of the system. We are allowed to choose an adapted control $y(\cdot)$ which drives the state $x(\cdot)$ of the system to the given target $X$ at time $t = 1$. This is so-called reachability problem. So in fact we are looking for a pair of stochastic processes $\{x(t), y(t), 0 \le t \le 1\}$ with values in $I\!R \times I\!R$ which is $\mathcal{F}_t$-adapted and satisfies the above equation. Such pair is called an adapted solution of the equation. Pardoux and Perng (see [13]) showed the existence and uniqueness of the adapted solution under the condition that $f(t, x, y)$ and $g(t, x)$ are uniformly Lipschitz continuous in $(x, y)$ or in $x$ respectively.