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ON AN EXPONENTIAL MARTINGALE APPROACH TO ALMOST SURE STABILITY OF ITÔ SDES IN $\mathbf{R}^1$

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Abstract. Almost sure stability of solutions to linear and nonlinear, nonautonomous stochastic differential equations (SDEs) driven by a standard Wiener process are discussed. For this purpose, we confine our study to SDEs with trivial equilibrium in $\mathbf{R}^1$ and exploit the well-known ideas of Dolean-Dade exponentials, martingale convergence and representation theorems, and Khinchin's law of iterated logarithm. Necessary and sufficient conditions of a.s. asymptotic stability of the trivial solution are obtained. All conditions are expressed in terms of coefficients of those equations. Eventually, we discuss polynomial decay rates to classify the asymptotic decay of its solutions.

Keywords. Stochastic differential equations, a.s. asymptotic stability, Dolean-Dade exponential, stochastic exponentials, martingale convergence theorems, law of iterated logarithm for stochastic integrals, polynomial decay rates.

AMS (MOS) subject classification: 34K50, 60H10, 93E15, 60G44, 60F15 .

1 Introduction

During a discussion of a.s. asymptotic stability of stochastic numerical methods we discovered an interesting technique combining the ideas of Dolean-Dade exponentials (see Protter [19] or Liptser and Shiryaev [13]), martingale convergence theorems (see Doob [5], Jacod and Protter [9], Liptser and Shiryaev [13], Shiryaev [25] and the law of iterated logarithm (see Karatzas and Shreve [10], Shiryaev [25]). For that particular discussion, see Rodkina and Schurz [21] and Schurz [22]. This technique turned out to be more efficient than the direct application of Itô formula (see Itô [8]) applied to moments of solutions of stochastic differential equations (SDEs) or stochastic difference equations for the purpose of asymptotic stability investigations. Independently, in Appleby [1] and Appleby and Reynolds [2] the strong law of large numbers for martingales (see Shiryaev [25]) have been exploited to discuss the exponential and nonexponential asymptotic stability of solutions