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A MASSERA THEORY OF IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. A Massera type theorem about impulsive differential equations is proved. The theorem is useful for the further study of the periodic solution problem with some type of discontinuities such as impulses.

Keywords. Massera theorem; periodic solution; impulsive differential equation

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1 Introduction and main result

Consider the first order differential equation

$$\frac{dx}{dt} = f(t, x), \qquad t \geq 0, \qquad x \in \mathbf{R}, \tag{1.1}$$

where $f \in C(\mathbf{R}^+ \times \mathbf{R}, \mathbf{R})(\mathbf{R}^+ = [0, \infty))$, and is ω-periodic on t. In 1950 Massera [1] proved the following theorem.

Theorem A. *Assume that for any* $(t_0, x_0) \in \mathbf{R}^+ \times \mathbf{R}$*, Eq.* (1.1) *has a unique solution for* $t \geq t_0$ *with* $x(t_0) = x_0$*. Then the existence of a positive bounded solution of* (1.1) *implies that Eq.* (1.1) *has at least one* ω*-periodic solution.*

It is well known that the Massera type theorem play an important role in the study of periodic solution problems of differential equations. In fact, the problem of establishing the existence of periodic solutions of differential equations via the boundedness of solutions has being the subject of many investigations since the Massera's work. See, for example, Massera [1], Hale [2], Yoshizawa [3] and Burton [4].

The purpose of this work is establishing a counterpart Massera type theorem for the impulsive differential equation of the form

$$\begin{cases} \frac{dx}{dt} = f(t, x), \quad t \geq 0, \quad t \neq t_k, \quad x \in \mathbf{R}, \\ \triangle x = I_k(x), \quad t = t_k, \quad k \in \mathbf{Z}^+, \end{cases} \tag{1.2}$$

where $f : \mathbf{R}^+ \times \mathbf{R} \to \mathbf{R}$ and is ω-periodic on t, $\mathbf{Z}^+$ denotes the set of positive integers, $I_k : \mathbf{R} \to \mathbf{R}, k \in \mathbf{Z}^+$, and $\{t_k\}$ is a known impulsive