

SUBNORMAL SOLUTIONS OF HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we investigate the growth of solutions and the existence of subnormal solutions of a class of higher order linear differential equations. We obtain some results which improve and extend the results of Chen-Shon [1], Qi-Yang [8] and Liu-Yang [7].

Keywords. Differential equations, subnormal solution, growth, entire functions, finite order.

1 Introduction and main results

In this paper a meromorphic function will mean meromorphic in the whole complex plane, and we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna Theory of meromorphic functions (e.g. see [10][11][12]). It is well known that the order $\sigma(f)$ and hyper-order $\sigma_2(f)$ of a meromorphic function f are defined by

$$\sigma(f) = \limsup_{r \rightarrow \infty} \frac{\log T(r, f)}{\log r},$$
$$\sigma_2(f) = \limsup_{r \rightarrow \infty} \frac{\log \log T(r, f)}{\log r}.$$

In the study of the solutions of complex differential equations, the growth of a solution is an important property. For linear differential equations of the form

$$f^{(n)} + a_{n-1}(z)f^{(n-1)} + \cdots + a_0(z)f = 0, \quad (1.0)$$

where $a(z)$, $a_0(z)$, \cdots , $a_{n-1}(z)$ are entire functions, it is well known that any solutions of the equation (1.0) must be entire functions of finite order if $a(z)$, $a_0(z)$, \cdots , $a_{n-1}(z)$ are polynomials; and if one among the coefficients of the equation (1.0) is transcendental, then the equation (1.0) has infinite order entire solutions (see [5]).