Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **18 (2011)** 341-352
 http://www.watam.org

EXTREMAL SOLUTIONS TO FIRST-ORDER DIFFERENTIAL INCLUSIONS UNDER WEAK ASSUMPTIONS

Daniel C. Biles[1] and Rodrigo López Pouso[2]

[1]Department of Mathematics, Belmont University,
1900 Belmont Blvd., Nashville, TN 37212, U.S.A.

[2]Departamento de Análise Matemática, Facultade de Matemáticas,
Universidade de Santiago de Compostela, Campus Sur,
15782 Santiago de Compostela, Spain.

Corresponding author email:rodrigo.lopez@usc.es

Abstract. In this paper we prove new existence results for first-order differential inclusions under weak conditions on the nonlinear part. We only place assumptions on the minimum (or maximum) values of the multifunctions, thus avoiding many of the usual assumptions such as closed values, convex values, monotonicity or (lower/upper) semicontinuity. One of the main keys is the existence of admissible non-quasisemicontinuity curves.

Keywords. Differential inclusions; semicontinuity; discontinuity.

AMS (MOS) subject classification: 34A12, 34A36, 34A60.

1 INTRODUCTION AND PRELIMINARIES

Let $T > 0$ be fixed and consider the real interval $I = [0, T]$ along with a multifunction $F : I \times \mathbb{R} \to P(\mathbb{R}) \setminus \{\emptyset\}$, where $P(\mathbb{R})$ stands for the set of all subsets of $\mathbb{R}$.

We consider the initial value problem

$$x'(t) \in F(t, x(t)) \quad \text{for almost all (a.a.) } t \in I, \quad x(0) = 0, \tag{1}$$

and our goal is to deduce mild conditions which imply that the least superfunction (or upper solution) is a solution of (1). This plan has been succesfully carried out several times to prove existence results for ordinary differential equations, see [9, 10, 11, 21, 23, 25]. At the sight of the results obtained in those references, this is possibly the most effective method for reducing assumptions on F. Interestingly, the existence results for (1) that we prove in this paper apply for multifunctions F which need not satisfy the usual hypotheses such as monotonicity or upper/lower semicontinuity. Moreover, F need not assume closed or convex values.

There is a noticeable recent interest on existence of solutions for differential inclusions under weak conditions, see for instance [12, 14, 24]. For recent papers on existence of solutions in connection with lower and upper