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SOME NONLINEAR CONTRACTION THEOREMS IN $\mathcal{L}$-FUZZY SPACES

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Abstract. In this paper first we prove a fixed point theorem in $\mathcal{L}$-fuzzy metric spaces. Next we prove a common fixed point theorem in $\mathcal{L}$-fuzzy metric spaces. Then we present the nonlinear contraction case of Jungck's common fixed point theorem in $\mathcal{L}$-fuzzy metric spaces. Finally, we introduce the concept of $\mathcal{L}$-fuzzy generalized distance on a $\mathcal{L}$-fuzzy metric space and prove a fixed point theorem.

Keywords. $\mathcal{L}$-fuzzy metric spaces; $\mathcal{L}$-fuzzy normed spaces; completeness; nonlinear contraction; fixed point theorem.

AMS (MOS) subject classification: 54E40; 54E35; 54H25

1 Introduction and Preliminaries

The notion of fuzzy sets was introduced by Zadeh [30]. Various concepts of fuzzy metric spaces were considered in [6, 25]. Many authors have studied fixed theory in fuzzy metric spaces; see [2, 3, 4, 7, 8, 12, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29].

In the sequel, we shall adopt the usual terminology, notation and conventions of $\mathcal{L}$-fuzzy metric spaces introduced by Saadati et al. [24].

Definition 1.1 ([7]) Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice, and U a non-empty set called universe. An $\mathcal{L}$-fuzzy set $\mathcal{A}$ on U is defined as a mapping $\mathcal{A} : U \longrightarrow L$. For each u in U, $\mathcal{A}(u)$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Classically, a triangular norm T on $([0,1], \leq)$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1, x) = x$, for all $x \in [0,1]$. These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (L, \leq_L)$. Define first $0_{\mathcal{L}} = \inf L$ and $1_{\mathcal{L}} = \sup L$.

Definition 1.2 ([11]) A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathcal{T} : L^2 \to L$ satisfying the following conditions:

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