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A BOUNDED HURWITZ VECTOR FIELD IN $\mathbb{R}^4$ HAVING A PERIODIC ORBIT

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Abstract. We modify the Bernat–Llibre Counterexample in order to obtain a smooth bounded vector field which satisfies the Markus–Yamabe hypotheses and has a periodic orbit.

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1 Introduction

Let $X : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1-vector field. Consider the differential system

$$\dot{x} = X(x)\,. \tag{1}$$

Let p be a singular point of X, that is, $X(p) = 0$. We say that p is a *global attractor* of the differential system (1) (or the vector field X) if $\phi(t, x)$ is defined for all $t > 0$ and tends to p as t tends to infinity for each $x \in \mathbb{R}^n$. Here $\phi(t, x)$ is the solution of (1) with initial condition $\phi(0, x) = x$.

In [6], L. Markus and H. Yamabe establish their well known global stability conjecture. For a simpler formulation of the Conjecture we consider the next Definition.

Definition 1.1. *Let $X : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1-vector field. We say that X is* **Hurwitz** *if for any $x \in \mathbb{R}^n$, all the eigenvalues of $JX(x)$ have negative real part. Here $JX(x)$ is the Jacobian of the map X at x.*

The Markus–Yamabe Conjecture (MYC). Let $X : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1-Hurwitz vector field. If $X(p) = 0$, then p is a global attractor of system (1).

As is well known, the Conjecture is only true for $n \leq 2$. For any $n \geq 3$, A. Cima et al. [4] give an example of a polynomial Hurwitz vector field of $\mathbb{R}^n$ which has orbits that scape to infinity. Moreover, a family of polynomial Hurwitz vector fields having orbits that scape to infinity is obtained in [5]; this family contains the preceding vector field.