Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **18 (2011)** 297-310
 http://www.watam.org

NEW PROXIMAL POINT ALGORITHMS BASED ON GENERALIZED YOSIDA APPROXIMATIONS APPLIED TO VARIATIONAL INCLUSIONS

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Abstract. A new class of relaxed proximal point algorithms involving Yosida approximations to the context of solving a general class of nonlinear variational inclusion problems based on the notion of maximal $(\eta)-$monotonicity is developed and examined. Convergence analysis turns out to be reasonable, and the sequence converges weakly to a unique solution of the variational inclusion problem. Furthermore, the algorithm developed in this communication seems to be appropriate to the generalized Yosida approximation in the sense that it can be applied to first-order evolution equations/inclusions as well.

Keywords. Proximal point algorithms; Yosida approximation; Nonlinear variational inclusion problems; Maximal $(\eta)-$monotonicity.

AMS subject classifications: 49J40; 65B05.

1 Introduction

Let X denote a real Hilbert space with the norm $\|\cdot\|$ and the inner product $\langle\cdot,\cdot\rangle$. We consider the nonlinear variational inclusion problem: determine a solution to

$$0 \in M(x), \tag{1}$$

where $M : X \to 2^X$ is a set-valued mapping on X.

Rockafellar [18] introduced and investigated a general version of the proximal point algorithm to the context of solving (1), while discussed the general convergence analysis for M maximal monotone. The proximal point algorithm for a sequence $\{x^k\}$ is generated for an initial point x^0 by

$$x^{k+1} \approx P_k(x^k) \tag{2}$$

such that it converges weakly to a solution of (1), provided the approximation is made sufficiently accurate as the iteration proceeds, where $P_k = (I + c_kM)^{-1}$ for a sequence $\{c_k\}$ of positive real numbers that is bounded away from zero is the resolvent of M. We observe from (2) that x^{k+1} is an approximate solution to inclusion problem

$$0 \in M(x) + c_k^{-1}(x - x^k). \tag{3}$$