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LEFSCHETZ FIXED POINT THEORY FOR PERMISSIBLE COMPACT ABSORBING TYPE CONTRACTIONS

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Abstract. Several new fixed point results for permissible self maps are presented in this paper. In particular we discuss compact absorbing contractions.

Keywords. Fixed point theory; Compact absorbing contractions; Permissible maps.

AMS (MOS) subject classification: 47H10.

1 Introduction

In Section 2 we present new results on fixed point theory for permissible maps which are compact absorbing contractions. In Section 3 we provide an alternative approach using projective limits. These results improve those in the literature; see [1-14] and the references therein. Our results were motivated in part from ideas in [7, 9, 12, 14].

Consider vector spaces over a field K. Let E be a vector space and $f : E \to E$ an endomorphism. Now let $N(f) = \{x \in E : f^{(n)}(x) = 0$ for some $n\}$ where $f^{(n)}$ is the n^{th} iterate of f, and let $\tilde{E} = E \setminus N(f)$. Since $f(N(f)) \subseteq N(f)$ we have the induced endomorphism $\tilde{f} : \tilde{E} \to \tilde{E}$. We call f admissible if $dim\, \tilde{E} < \infty$; for such f we define the generalized trace $Tr(f)$ of f by putting $Tr(f) = tr(\tilde{f})$ where tr stands for the ordinary trace.

Let $f = \{f_q\} : E \to E$ be an endomorphism of degree zero of a graded vector space $E = \{E_q\}$. We call f a Leray endomorphism if (i). all f_q are admissible and (ii). almost all $\tilde{E}_q$ are trivial. For such f we define the generalized Lefschetz number $\Lambda(f)$ by

$$\Lambda(f) = \sum_q (-1)^q \, Tr\,(f_q).$$

A linear map $f : E \to E$ of a vector space E into itself is called weakly nilpotent provided for every $x \in E$ there exists n_x such that $f^{n_x}(x) = 0$.

Assume that $E = \{E_q\}$ is a graded vector space and $f = \{f_q\} : E \to E$ is an endomorphism. We say that f is weakly nilpotent iff f_q is weakly nilpotent for every q.