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POSTERIORI ANALYSIS OF UNSTEADY NAVIER-STOKES EQUATIONS WITH THE CORIOLIS FORCE

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Abstract. We present a posteriori error analysis for the unsteady Navier-Stokes equations with the coriolic force. Our analysis covers the Galerkin finite element approximation in space and the Euler full-implicit scheme in time. This scheme is implicit for the linear and nonlinear terms. For the discretization, we propose and analyse two types of error indicators, with one being for the time discretization and the other for the space discretization. Finally, we prove the equivalence between the sum of the two types of error indicators and the full error, in order to work with adaptive time steps and finite element meshes.

Keywords. Navier-Stokes equations; Fully discrete approximations; A posteriori error estimates; Finite element; Coriolis force; Error indicator.

AMS (MOS) subject classification: 65J15; 65N15; 65N30; 76D05.

1 Introduction

Let Ω be a bounded domain in R^2 assumed to have a Lipschitz continuous boundary $\partial\Omega$ and to satisfy a further condition stated in (A1) below. We consider the unsteady Navier-Stokes equations with the coriolis force

$$\begin{cases} \partial_t \mathbf{u} - \nu\triangle\mathbf{u} + \omega\times\mathbf{u} + \nabla p + (\mathbf{u}\cdot\nabla)\mathbf{u} = \mathbf{f} & \text{in } \Omega\times(0,T], \\ \mathrm{div}\mathbf{u} = 0 & \text{in } \Omega\times(0,T], \\ \mathbf{u} = \mathbf{0} & \text{on } \partial\Omega\times[0,T], \\ \mathbf{u}(x,0) = \mathbf{u}_0(x) & \text{in } \Omega, \end{cases} \tag{1}$$

where $\nu > 0$ is the viscosity, $T > 0$ represents a finite time. $\omega\times\mathbf{u} = (-\omega u_2, \omega u_1)^T$ is the coriolis force, where ω is the angular velocity of rotation of the frame of reference and $\mathbf{u}$ is the velocity field referred to this rotating reference. p is the pressure, $\mathbf{f}$ includes the external force and the centrifugal force, $\mathbf{u}_0(x)$ is the initial velocity. For simplicity, we will denote $\mathbf{u}$ as u.

There are numerous works devoted to the development of efficient schemes for the Navier-Stokes equations(see [1-7] for instance), which include a priori error analysis and a posteriori error analysis. Recently, a new a posteriori error analysis, based on backward Euler's scheme in time and conforming or